$$+f_\nu^{(2m+1)}(v)\begin{Bmatrix}\cos kv\\ \sin kv\end{Bmatrix}\Bigg) \tag{18}$$

where

$$f_\nu^{(n)}(v)=\frac{d^n}{dv^n}(v^2+r^2)^{-(2\nu+1)/2} \tag{19}$$

then asymptotic expansions of the functions $B_R^{(\nu)}$ and $B_I^{(\nu)}$ can be obtained by integrating by parts as

$$B_R^{(\nu)} \sim F_c^\nu(X)+c \tag{20}$$

$$B_I^{(\nu)} \sim F_s^\nu(X) \tag{21}$$

where c is an integration constant,

$$c=\frac{2}{(2\nu-1)!!}\left(\frac{k}{r}\right)^\nu K_\nu(kr) \qquad X>0$$

$$=0 \qquad X<0 \tag{22}$$

The derivative, Eq. (19), can be calculated by the recursion formula

$$(v^2+r^2)f_\nu^{(n+2)}+(2\nu+2n+3)vf_\nu^{(n+1)}$$
$$+(n+1)(2\nu+n+1)f_\nu^{(n)}=0 \tag{23}$$

Concluding Remarks

Appropriate series for numerical computation of the integral function Eqs. (1) and (2) have been obtained as Eqs. (6) and (7). These series hold except in the true singular domain $r=0$, $X\geq 0$ for $B_R^{(\nu)}$, and $r=X=0$ for $B_I^{(\nu)}$. Even in the proximity of singularities, however, the series are tractable because every singular part occurs explicitly in the initial terms, which is a favorable characteristic if one considers their application to the aerodynamic theory. It is worthy of notice that only the first of the two summations in $U_m^{(\nu)}$ is needed for Eqs. (6) and (7) if the lower limit of the integral Eqs. (1) and (2) is finite as is the case for supersonic flow.

The asymptotic expansions (20) and (21) can be used for large arguments. Numerical examples for $\nu=1$ are found in Ref. 8.

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Evolution of Swirl in Two-Dimensional-Nozzle Flow

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In a typical turbine engine, the residual swirl in the exhaust gases, though somewhat detrimental to performance, has been recognized as a mechanism that favors mixing of the exhaust.[1,2] In an axisymmetric nozzle plume, increased mixing due to shear tends to lower the plume temperature somewhat and hence contributes to some reduction of the jet noise and IR signature. As interest in two-dimensional (2D) nozzles continues to rise, some questions about the behavior of engine swirl in 2D nozzles are raised: What happens to the engine swirl going through a rectangular flow passage? And how does this affect mixing in the plume? This Note reports some simple studies in Northrop's water tunnel[3] which provide preliminary answers to these questions.

In the water-tunnel experiments, a plastic duct (Fig. 1) was aligned with the flow in the center of the water tunnel. Water entered through a circular bellmouth and duct and then passed to a rectangular nozzle via a transition section. The exit area was made equal to the circular area to avoid separation. One or more plastic pinwheels with blade angles up to 22.5 deg were placed upstream in the circular section to produce a swirling flow. Flow paths were observed through dyes released from tubes within the duct and from probes in the external flow outside of the nozzle. Two models with rectangular exits of aspects ratios (Æ) 4 and 10 were tested. The tunnel velocity was approximately 0.7 ft/s. The pressure losses produced by the pinwheel in this arrangement gave somewhat lower nozzle exit velocities than those in the external flow; however, the phenomena observed are expected to be typical for jets of higher velocities. For clarity, results are presented for a blade angle of 22.5 deg, which was the maximum tested.

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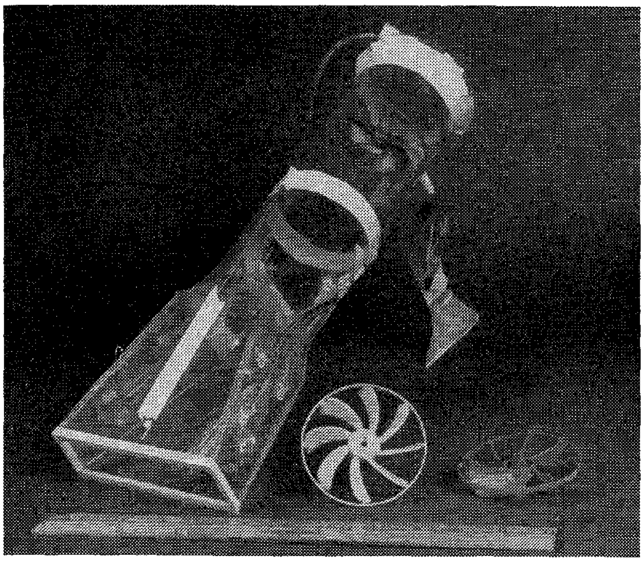

Fig. 1 4:1 aspect ratio nozzle and duct and pinwheels.

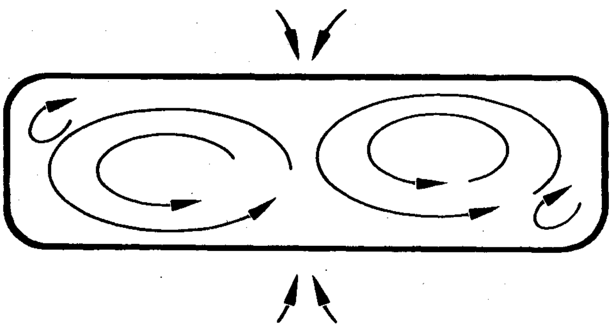

Fig. 2 Sketch of time-averaged vortex structure near the exit of the 4:1 aspect ratio nozzle.

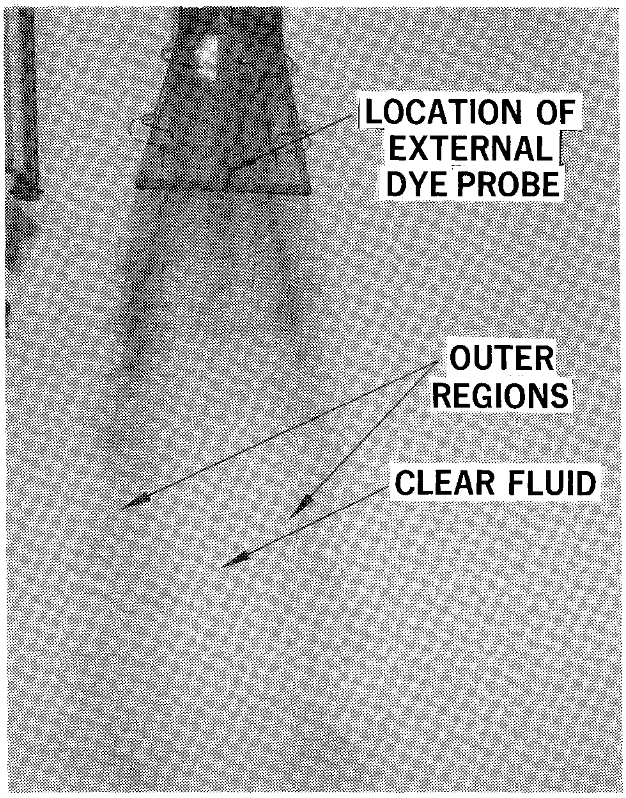

Fig. 3 Jet from the 4:1 aspect ratio nozzle, broadside-on.

In all cases, the flow downstream of the pinwheels contained large-scale turbulent eddies which destroyed the filament-like structure of the dye and so prevented vortical motions from being recorded on still photographs. Nonetheless, major swirling and other motions of the time-averaged flow could be clearly observed by eye. The phenomena were complex and more elaborate experiments are urgently needed. The following is an interpretation of our observations to date.

In the case of the 4:1 exit aspect ratio nozzle, the induced swirl occupied the whole of the circular passage and the majority of the rectangular cross section. Although direct observations were difficult because of the large turbulent eddies in the nozzle, it appeared that the main swirl had split up into two corotating vortices in the vicinity of the exit. In addition, small but distinct contrarotating vortices were induced in two of the corners; the particular corners were probably determined by the direction of the swirl. Figure 2 is a sketch of the apparent vortex structure at a cross section just after the exit. The splitting of the main swirl is supported by the most striking feature of the jet: The dyed fluid coming from within the nozzle was almost completely confined to the two outer regions when viewing the jet broadside-on (Fig. 3), and the central region appeared to consist of clear fluid coming from the external flow. This phenomenon of forced entrainment was confirmed by means of dye from two probes located outside the nozzle (see Fig. 3 for the probe location). Figure 4 is a view taken narrow-end-on. It shows the external dye being strongly swept into the center of the jet from both sides almost equally. In the outer regions of the jet (where fluid had originated in the nozzle), rotation in the main swirl direction could be detected for some distance downstream although it was heavily damped by the entrainment process. These experiments were repeated without the pinwheels. The flow went straight through the nozzle and no persistent vortex motions were observed. The external dye mixed much more slowly with the jet through parallel turbulent mixing along the sides (see Fig. 5).

The pair of corotating vortices promotes forced mixing through large exchanges of fluid between the outer and core regions of the jet. Outside the nozzle, these vortices draw large quantities of external flow into the jet. This phenomenon is thus responsible for a powerful mixing mechanism which is not present in a swirling axisymmetric jet or in a nonswirling rectangular jet.

The major difference observed with the nozzle of Æ 10:1 was the greater prominence of the contrarotating vortices. Within the nozzle, each of the two vortices occupied roughly the outer quarter of the cross section and were not noticeably biased towards the corners. These vortices gave rise to a different structure in the jet also. The dye from the nozzle was observed to occupy three regions (rather than two) with clear external fluid between them. The outer regions consisted largely of flow from the contrarotating vortices and this motion persisted somewhat weakly in the jet. The central region of the jet emanated from the main swirl within the nozzle but, again, the strength of the swirl in the jet was much reduced. It thus appears that, for this high aspect ratio nozzle, the main swirl does not subdivide but acts with the contrarotating vortices to draw external fluid into the jet. This will again have a major impact on the jet characteristics.

Perhaps some caution is necessary in comparing the flows in these two nozzles since they differ not only in final aspect ratio but also in length (the 10:1 Æ nozzle has a length of 4.33 duct-entry diameters compared with 2.3 for the 4:1 Æ nozzle). The contrarotating vortices in the 10:1 Æ nozzle continue to develop within the whole of the transition section and it is possible that a longer 4:1 Æ nozzle would produce stronger contrarotating vortices than those observed.

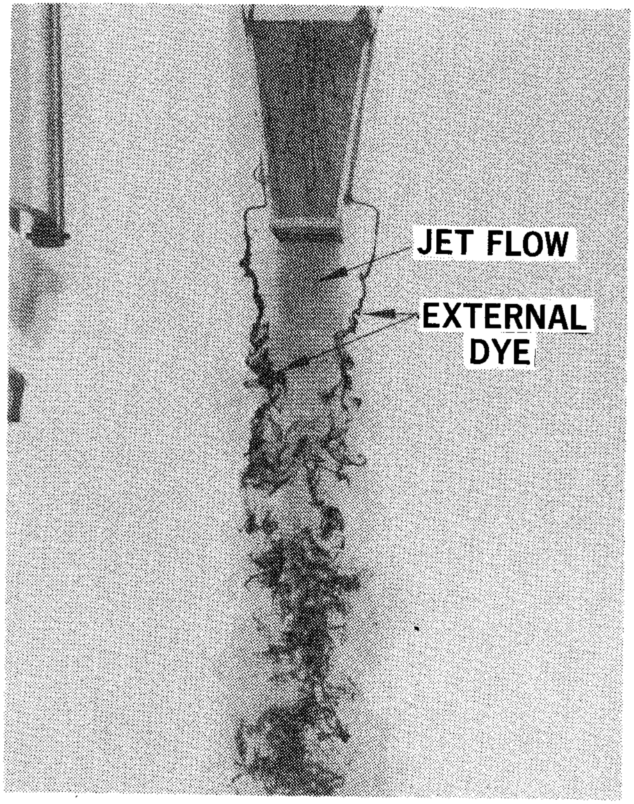

Fig. 4 Jet from the 4:1 aspect ratio nozzle, narrow-end-on, showing external fluid being drawn into the jet.

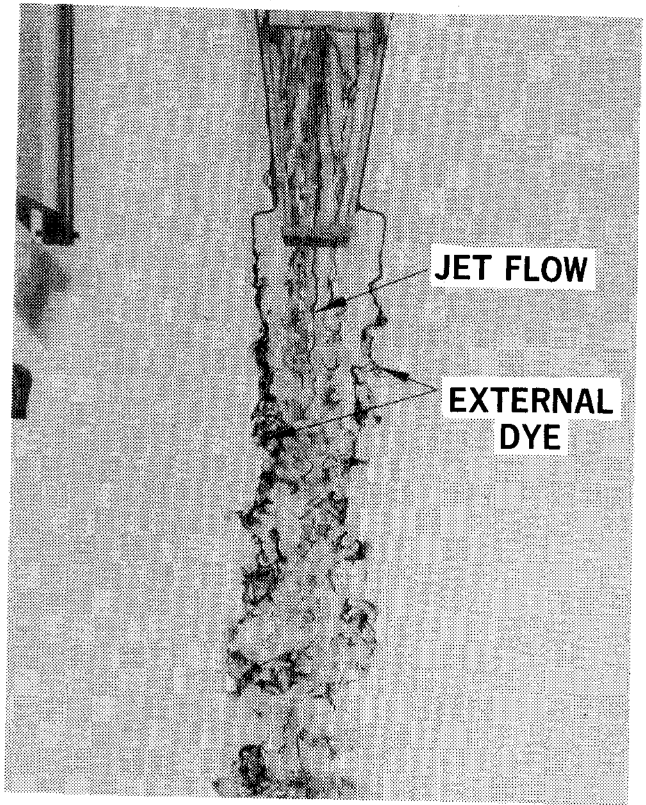

Fig. 5 Jet from the 4:1 aspect ratio nozzle without pinwheels.

There is clearly much to be learned about the effect of swirl on the flow within rectangular nozzles and on the emerging jet; however, evidence has been collected here to show that swirl is likely to result in the large-scale interchange of hot exhaust gases with colder gases and with the cold external flow, and hence to have a major impact on the plume temperature and IR signature. A simple, approximate model of the effect has been developed and applied in Ref. 4.

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